# EXERCISE – IV HINTS & SOLUTIONS

**Sol.1** $3\vec{p} = (3x + 12y)\,\vec{a} + (6x + 3y + 3)\,\vec{b}$

$2\vec{q} = (2y - 4x + 4)\,\vec{a} + (4x - 6y - 2)\,\vec{b}$

$3\vec{p} = 2\vec{q} \Rightarrow 3x + 12y = -4x + 2y + 4$

$7x + 10y = 4$ ....(1)

and $6x + 3y + 3 = 4x - 6y - 2$

$2x + 9y = -5$ ....(2)

Solving equation 1 and 2

$x = 2,\ y = 1$

**Sol.2** **(a)** Let $\vec{P}, \vec{Q}, \& \vec{R}$ be three vectors

$\overrightarrow{PQ} = \vec{a} + 5\vec{b} - 7\vec{c}$

$\overrightarrow{QR} = -2\vec{a} - 10\vec{b} + 14\vec{c}$

$(\overrightarrow{QR}) = -2(\overrightarrow{PQ})$

Here collinear.

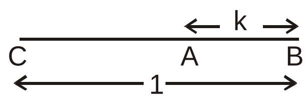
**(b)** $\overrightarrow{AB} = (2, 2, 4)$

$\overrightarrow{AC} = (-6, -6, -12) \Rightarrow \overrightarrow{AC} = -3\overrightarrow{AB}$

Here $\vec{A}, \vec{B}, \vec{C}$ are collinear

Let the ratio be k : 1

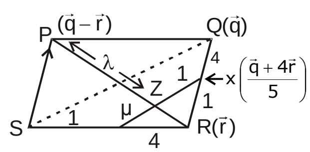
So,




$\dfrac{-3k-1}{k-1} = 3 \Rightarrow k = \dfrac{1}{3}$ (externally)

**Sol.3** P.V. of Z are :

$$\frac{\mu\vec{q} + \dfrac{4\mu\vec{r}}{5} + \dfrac{\vec{r}}{5}}{\mu+1} = \frac{\lambda\vec{r} + \vec{q} - \vec{r}}{\lambda+1}$$




$\dfrac{\mu}{5(\mu+1)} = \dfrac{1}{\lambda+1}$ and $\dfrac{4\mu+1}{5(\mu+1)} = \dfrac{\lambda-1}{\lambda+1}$

$\dfrac{\mu}{4(\mu+1)} = \dfrac{1}{\lambda-1} \Rightarrow \lambda = \dfrac{5\mu+1}{\mu}$

$\mu = 4,\ \lambda = \dfrac{21}{4} \Rightarrow \dfrac{\overrightarrow{PZ}}{\overrightarrow{PR}} = \dfrac{\lambda}{\lambda+1} = \dfrac{21}{25}$

**Sol.4** **(i)** $\vec{r}_2 = (2, 1, 3) - 2\mu\,(3, -2, 4)$

Here both lines are parallel.

**(ii)** $1 + \lambda = 2 + 2\mu$ ....(1)

$-1 - \lambda = 4 + \mu$ ....(2)

$3 + \lambda = 6 + 3\mu$ ....(3)

from (1) and (2)

$3\mu = -6 \Rightarrow \mu = -2$

then $\lambda = -3$

Since $\lambda = -3$ and $\mu = -2$

satisfies (3), hence both lines are intersecting.

**(iii)** $1 + \lambda = 2 + 4\mu$ ...(1)

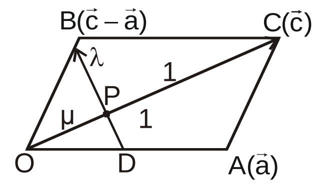
$3\lambda = 3 - \mu$ ...(2)

$1 + 4\lambda = \mu$ ...(3)

from 2 and 3 $7\lambda + 1 = 3 \Rightarrow \lambda = \dfrac{2}{7},\ \mu = \dfrac{15}{7}$

since these values of $\lambda$ and $\mu$ not satisfy 1 hence non-intersecting.

**Sol.5**




P.V. of P are $\dfrac{\left(\dfrac{\lambda}{2}-1\right)\vec{a} + \vec{c}}{\lambda+1} = \dfrac{\mu\vec{c}}{\mu+1}$

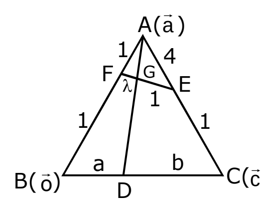
$\lambda = 2,\ \ \mu = \dfrac{1}{2}$

Here BD and CO intersect in the same ratio.

**Sol.6** P.V. of 'E' $= \dfrac{(\vec{a} + 4\vec{c})}{5}$

Now let FG : GE = $\ell$ : 1

So. P.V. of G are:




$$\frac{\left(\dfrac{3a}{a+b}\right)\vec{c} + 2\vec{a}}{5} = \frac{\dfrac{\lambda}{5}(\vec{a} + 4\vec{c}) + \dfrac{\vec{a}}{2}}{\lambda+1}$$

comparing coefficient of $\vec{a}, \lambda = \dfrac{1}{2}$

comparing coefficient of $\vec{c}$, $\dfrac{4}{15} = \dfrac{3a}{5(a+b)}$

$\dfrac{a}{b} = \dfrac{4}{5} \Rightarrow a + b = 9$

**Sol.7** $(\vec{a} + \vec{b}).\vec{a} = 0$

$a^2 + \vec{a}.\vec{b} = 0 \Rightarrow \vec{a}.\vec{b} = a^2$ ..... (1)

$(2\vec{a} + \vec{b}).\vec{b} = 0$

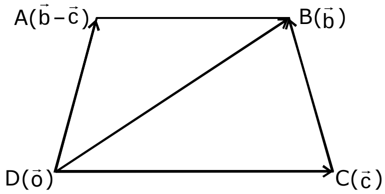
$2\vec{a}.\vec{b} + |\vec{b}|^2 = 0$

$-2a^2 + 2a^2 = 0$ Hence proved.

**Sol.8** $\overrightarrow{DA} = \overrightarrow{CB}$

$\overrightarrow{DA} = (\vec{b} - \vec{c})$

In $\Delta$DAC, $\overrightarrow{DA} + \overrightarrow{AC} = \overrightarrow{DC}$




$\overrightarrow{AC} = 2\vec{c} - \vec{b}$

In $\Delta$DBC, $\overrightarrow{DB} + \overrightarrow{BC} = \overrightarrow{DC}$

$\overrightarrow{DB} = \vec{c} - \overrightarrow{BC}$

$\overrightarrow{DB} = \vec{c} - \vec{b} - \vec{c} - 2\vec{c} - \vec{b}$ **Hence proved.**

**Sol.9** (1 – x, –y) . (– 1 – x, – y)
– 1 + $x^2$ + $y^2$ – 3 = 0
$x^2 + y^2 = 4$
$x^2 \in [0, 4]$

$\sqrt{(1-x)^2+y^2}\ \ \sqrt{(1+x)^2+y^2}$

$\sqrt{5-2x}\ \ \sqrt{5+2x}$

$\sqrt{25-4x^2}$

M = $\sqrt{25-4\times4}$ = $\sqrt{9}$

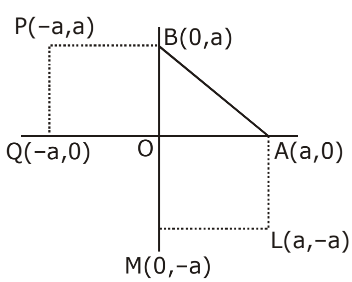
M = $\sqrt{25-4\times0}$ = $\sqrt{25}$

$M^2 + m^2 = 25 + 9 = 34$

**Sol.10** $\overrightarrow{AW}\cdot\overrightarrow{BX}$ = $(\vec{x}-\vec{a})\cdot(\vec{b}-\vec{y})$

= $\vec{x}\cdot\vec{b}$ – $\vec{x}\cdot\vec{y}$ $\vec{a}\cdot\vec{b}$ + $\vec{a}\cdot\vec{y}$ = 0 = RHS.

**Sol.11** Let O be origen.
AP 2x + y – a = 0
BL x + 2y – a = 0




point of intersection of AP & BL & let P $\left(\frac{a}{3}, \frac{a}{3}\right)$

equation or line $\perp$ to AB & through origen y = x .
so it is clear that P lie on line y = x.

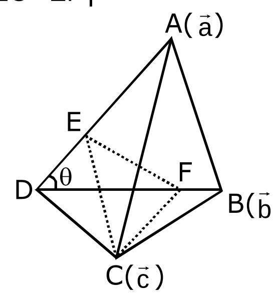
**Sol.12** Let $\vec{R}$ = (x, y, z)

$(x-2y+3z-10)\hat{i}+(2x+y+4z-20)\hat{j}+(x+3y+3z-20)\hat{k}=0$

x–2y+3z=10, 2x + y + 4z = 20, x + 3y + 3z = 20
On solving: x = – 1, y = 2, z = 5

So, $\vec{R} = -\hat{i} + 2\hat{j} + 5\hat{k}$

**Sol.13** Required area = $\frac{1}{2}|\overrightarrow{EC}\times\overrightarrow{EF}|$




$= \frac{1}{2}\left|\left(\vec{c}-\frac{2\vec{a}}{3}\right)\times\left(\frac{\vec{b}-2\vec{a}}{3}\right)\right|$

$= \frac{1}{2}\left|\frac{\vec{c}\times\vec{b}}{3}+\frac{2\vec{a}\times\vec{c}}{3}+\frac{\vec{b}\times2\vec{a}}{9}\right|$

$= \frac{1}{2}\left|\left(\frac{a^2\sin\alpha}{3}\right)\hat{\eta}_1+\left(\frac{2a^2\sin\alpha}{3}\right)\hat{\eta}_2+\left(\frac{2a^2\sin\alpha}{9}\right)\hat{\eta}_3\right|$

$= \frac{a^2}{6}\left|(\sin\alpha)\hat{\eta}_1+(2\sin\alpha)\hat{\eta}_2+\left(\frac{2}{3}\sin\alpha\right)\hat{\eta}_3\right|$

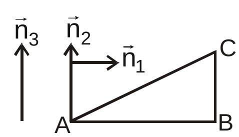
$= \frac{a^2}{6}\sqrt{\sin^2\alpha+4\sin^2\alpha+\frac{4}{9}\sin^2\alpha}$

$\frac{a^2}{6}\times\sqrt{\frac{49\sin^2\alpha}{9}}$ , $\sin^2\alpha = \frac{3}{4}$

$\frac{7a^2}{18}\sin\alpha \Rightarrow \frac{7a^2}{18}\left(\frac{\sqrt{3}}{2}\right) = \frac{7a^2}{12\sqrt{3}}$ sq. units.

**Sol.15** $\vec{n}_2$ = $\overrightarrow{AB}\times\overrightarrow{AC}$ = (3, –1, – 2)

$\vec{n}_3$ = (1, 0, 1 )




$\hat{n}_1 = \pm\frac{\vec{n}_2\times\vec{n}_3}{|\vec{n}_2\times\vec{n}_3|}$

$\hat{n}_1 = \pm\frac{1}{3\sqrt{3}}$ (–1, –5, 1)

**Sol.16** $\begin{vmatrix}(a_1-a)^2 & (a_1-b)^2 & (a_1-c)^2\\(b_1-a)^2 & (b_1-b)^2 & (b_1-c)^2\\(c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2\end{vmatrix} = 0$

$R_1 \to R_1 - R_2 \quad R_2 \to R_2 - R_3$

$$\begin{vmatrix} (a_1-b_1)(a_1+b_1-2a) & (a_1-b_1)(a_1+b_1-2b) & (a_1-b_1)(a_1+b_1-2c) \\ (b_1-c_1)(b_1+c_1-2a) & (b_1-c_1)(b_1+c_1-2b) & (b_1-c_1)(b_1+c_1-2c) \\ (c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2 \end{vmatrix} = 0$$

$$(a_1-b_1)\,(b_1-c_1)\begin{vmatrix} (a_1+b_1-2a) & (a_1+b_1-2b) & (a_1+b_1-2c) \\ (b_1+c_1-2a) & (b_1+c_1-2b) & (b_1+c_1-2c) \\ (c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2 \quad \& \quad C_2 \to C_2 - C_3$

$$(a_1-b_1)\,(b_1-c_1)\begin{vmatrix} 2(b-a) & 2(c-b) & a_1+b_1-2c \\ 2(b-a) & 2(c-b) & b_1+c_1-2c \\ (b-a)(2c_1-a-b) & (c-b)(2c_1-b-c) & (c_1-c)^2 \end{vmatrix} = 0$$

$$(a_1-b_1)(b_1-c_1)(b-a)(c-b)\begin{vmatrix} 2 & 2 & a_1+b_1-2c \\ 2 & 2 & b_1+c_1-2c \\ 2c_1-a-b & 2c_1-b-c & (c_1-c)^2 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$

$(a_1 - b_1)\,(b_1 - c_1)\,(b - a)\,(c - b)\,(a - b)\,(c_1 - a_1) = 0$ ...(i)

Given $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \Rightarrow (a - b)\,(b - c)\,(c - a) \neq 0$

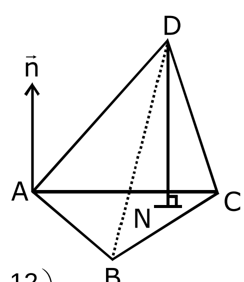
So from (i) $(a_1 - b_1)\,(b_1 - c_1)\,(c_1 - a_1) = 0$

$$\Rightarrow \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & b_1 & b_1^2 \\ 1 & c_1 & c_1^2 \end{vmatrix} = 0$$

**Sol.17** **(i)** Equation of BC

$\vec{r} = (3, 0, 1) + \lambda\,(1, 3, 5)$

$\overrightarrow{AM} \perp \overrightarrow{BC}$




$\Rightarrow \overrightarrow{AM}.\overrightarrow{BC} = 0$

$\lambda = \frac{1}{7}$,

So, P.V. of M $\Rightarrow \left(\frac{22}{7}, \frac{3}{7}, \frac{12}{7}\right)$

$|\overrightarrow{AM}| = \frac{6}{7}\sqrt{14}$

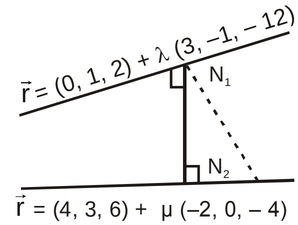
**(ii)** $V = \frac{1}{6}\left[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}\right] = \frac{1}{6}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = 6$

**(iii)** From figure $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = -2(\hat{i} + 8\hat{j} - 5\hat{k})$

Equation of DN : $\vec{r} = (2, 3, 2) + \mu\,(1, 8, -5)$

$\overrightarrow{AN} \perp \vec{n} \Rightarrow \mu = -\frac{1}{5}$ So, $\overrightarrow{DN} = \frac{3}{5}\sqrt{10}$

**(iv)**

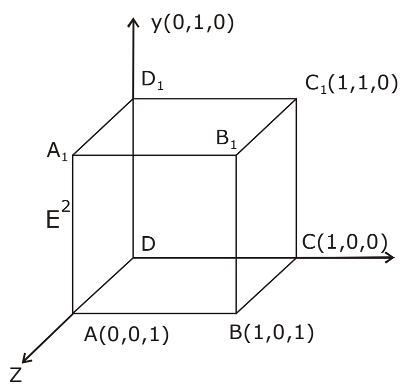



$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$d = \left|\frac{36}{\sqrt{216}}\right| = \sqrt{6}$$

**Sol.18** Let coordinate axes be three co=terminus edge of cube. So coordinate of A,B,C,D,A, B,C,D will be as shown is diagram

Now $\because$ $|\overrightarrow{AE}| = 1/3 \Rightarrow$ So $\frac{EA}{EA_1} = \frac{1}{2}$




P.V. of E $\left(0, \frac{1}{3}, 1\right)$

again $|\overrightarrow{BF}| = 1/4 \Rightarrow \frac{FB}{FC} = \frac{1}{3}$

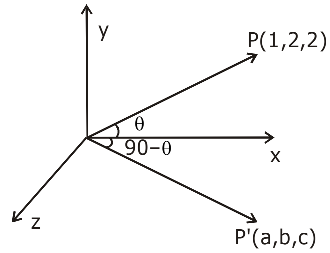
so P.V. of F $\left(1, 0, \frac{3}{4}\right)$

equation of plane OEF will be $\vec{r}\,.\,(5\hat{i} + 9\hat{j} + 8\hat{k})$

so distance of B (1, 1, 1) from plane OEF

$$= \left|\frac{5 + 9 + 8 - 11}{\sqrt{25 + 81 + 64}}\right| = \frac{11}{\sqrt{170}}$$

**Sol.19** $\cos\theta = \frac{1}{3}$




$\cos(90 - \theta) = \frac{a}{3}$

$\sin\theta = \frac{a}{3}$

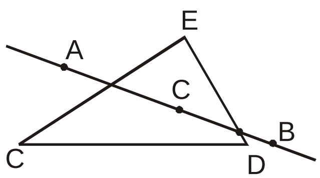
$\frac{2\sqrt{2}}{3} = \frac{a}{3} \quad \Rightarrow a = 2\sqrt{2}$

$a^2 + b^2 + c^2 = 3$

$a + 2b + 2c = 0$

Solve & get b & c

**Sol.20**




Equation of plane CDE : -

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\vec{n} = \overrightarrow{CD} \times \overrightarrow{CE} = 6\hat{i} - 4\hat{j} + 2\hat{k}$

$\vec{r} \cdot (6, -4, 2) = 24 \Rightarrow 3x - 2y + z = 12$

Equation of line AB :

$\vec{r} = (1, 2, 1) + \lambda\,(1, -1, 1)$

P.V. of 'R' : $1 + \lambda,\ 2 - \lambda,\ 1 + \lambda$

lies on plane CDE

$3(1 + \lambda) - 2(2 - \lambda) + 1 + \lambda = 12 \Rightarrow \lambda = 2$

P.V. of R : $(3\hat{i} + 3\hat{k})$

**Sol.21** $(n\vec{a} + \vec{b}).\{(n\vec{b} + \vec{c}) \times (n\vec{c} + \vec{a})\}$

$= (n\vec{a} + \vec{b}).\{n^2(\vec{b} \times \vec{c}) \times n(\vec{b} \times \vec{a}) + \vec{c} \times \vec{a}\}$

$n^3\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] + \left[\vec{b}\ \ \vec{c}\ \ \vec{a}\right]$

$(n^3 + 1)\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = (n^3 + 1)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$(n^3 + 1)\begin{vmatrix} \vec{a}.\hat{i} & \vec{a}.\hat{j} & \vec{a}.\hat{k} \\ \vec{b}.\hat{i} & \vec{b}.\hat{j} & \vec{b}.\hat{k} \\ \vec{c}.\hat{i} & \vec{c}.\hat{j} & \vec{c}.\hat{k} \end{vmatrix}$

**Sol.22** **(A)** R.H.S. $\sqrt{-\vec{b}.\{(\vec{a}.\vec{b})\vec{a} - a^2\vec{b}\}}$

$\Rightarrow \sqrt{-\vec{b}.\{(ab\cos\theta)\vec{a} - a^2\vec{b}\}}$

$\Rightarrow \sqrt{a^2b^2(1 - \cos^2\theta)} = ab\sin\theta$

$= |\vec{a} \times \vec{b}| \qquad = $ L.H.S.

**(B)** LHS. $|(\vec{a}\,.\,\vec{q})\vec{p} - (\vec{p}\,.\,\vec{q})\vec{a}|$

from $\vec{a} + \vec{b} = \mu\,\vec{p}$ ....(1)

$\vec{a}\,.\,\vec{q} = \mu\ \vec{p}\,.\,\vec{q}$ ....(2)

$\Rightarrow \left|(\mu\,\vec{p}\,.\,\vec{q})\vec{p} - (\vec{p}\,.\,\vec{q})\vec{a}\right| \Rightarrow \left|(\vec{p}\,.\,\vec{q})(\mu\,\vec{p}\,.\,\vec{q})\right|$

$\Rightarrow \left|(\vec{p}\,.\,\vec{q})\,\vec{b}\right| = |\vec{p}\,.\,\vec{q}| = $ R.H.S. $\quad (\because\ |\vec{b}|^2 = 1)$

**Sol.23** $(\vec{a}\,.\,\vec{c})\,\vec{b} - (\vec{a}\,.\,\vec{b})\,\vec{c} + (\vec{a}\,.\,\vec{b})\,\vec{b}$

$= (4 - 2\beta - \sin\alpha)\ \vec{b} + (\beta^2 - 1)\ \vec{c}$

Now from $(\vec{c}\,.\,\vec{c})\,\vec{a} = \vec{c}$

dot with $\vec{c}$ :

$c^2\ (\vec{a}\,.\,\vec{c}) = c^2 \Rightarrow \vec{a}\,.\,\vec{c} = 1$

Since $\vec{b}\ \&\ \vec{c}$ are non collinear and non-zero vectors, hence

$1 + \vec{a}\,.\,\vec{b} = 4 - 2\beta - \sin\alpha,\ -\ \vec{a}\,.\,\vec{b} = \beta^2 - 1$

Adding: $\beta^2 - 2\beta + 2 = \sin\alpha$

$(\beta - 1)^2 + (1 - \sin\alpha) = 0$, it is possible if $\beta = 1$ and $\sin\alpha = 1$

$\alpha = n\pi + (-1)^n\ \frac{\pi}{2}$

**Sol.24** P and Q are intersecting points of diagonals of parallelogram ABDE and ABCF.

P.V. of E $\Rightarrow (-7, 22, -1)$

P.V. of F $\Rightarrow (-2, 23, 4)$

Area of $\Delta AEF = \frac{1}{2}\ |\overrightarrow{AF} \times \overrightarrow{AE}|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ -2 & 0 & -6 \end{vmatrix} \qquad = \frac{1}{2}\left|-6\hat{i} + 20\hat{j} + 2\hat{k}\right|$

$= \frac{\sqrt{440}}{2} = \sqrt{110} = \sqrt{S} = S = 110$

**Sol.25** $\left[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})\right]\times(\vec{B}\times\vec{C})(\vec{B}+\vec{C})$

$\Rightarrow \vec{A}\vec{B}\vec{C}(C^2-B^2)=0 \quad (\because |\vec{C}|=|\vec{B}|)$

**Sol.26** $(\vec{a}\times\vec{b})\times\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\} = 0$

$(\vec{a}\times\vec{b})\times\{(\vec{b}\times\vec{c}).\vec{a})\,\vec{c}-(\vec{b}\times\vec{c}).\vec{c}\,)\,\vec{a}\}=0$

$(\vec{a}\times\vec{b})\times\{[\vec{b}\ \vec{c}\ \vec{a}]\,\vec{c}\} = 0$

$\{(\vec{a}\times\vec{b})\times\vec{c}\}\,[\vec{b}\ \vec{c}\ \vec{a}] = 0$

so $[\vec{b}\ \vec{c}\ \vec{a}] = 0$ or $(\vec{a}\times\vec{b})\times\vec{c} = 0$

$\begin{vmatrix}1 & 2\alpha & -2\\ 2 & -\alpha & 1\\ \alpha & 2 & -3\end{vmatrix} = 0$ or $(\vec{a}\times\vec{b})\times\vec{c} = 0$

$\alpha = 2/3$ or $-3\,(1,2\alpha,-2)-(2-4\alpha^2-2\alpha)(\alpha,2,-3)=0$
or $-3(1,2\alpha,-3)=(2-4\alpha^2-2\alpha)(\alpha,2,-3)=0$
no value or $\alpha$.

**Sol.27** Let $\vec{V} = x\hat{i}+y\hat{j}+z\hat{k}$

Given $\begin{vmatrix}x & y & z\\ 1 & 1 & -2\\ 1 & -2 & 1\end{vmatrix} = 0$

$= x + y + z = 0$ ......(1)

Also $\vec{V}\,.\,(-2\hat{i}+\hat{j}+\hat{k}) = 0$

$-2x + y + z = 0$ .... (2)

from equation (1) and (2)

$x = 0,\ y = -z$

Also given that : $\dfrac{\vec{V}\,.\,(1,-1,1)}{\sqrt{3}} = 6\sqrt{3}$

$x - y + z = 18$ ....(3)

Put $y = -z$

$y = -9$ and $z = 9$ so, $\vec{V} = -9\,(\hat{j}-\hat{k})$

**So.28** $\vec{d} = x\vec{a}+y\vec{b}+z\vec{c}$ ....(1)

Dot with $(\vec{b}\times\vec{c}),(\vec{c}\times\vec{a})\,\&\,(\vec{a}\times\vec{b})$ one by one:

$x = \dfrac{[\vec{b}\ \vec{c}\ \vec{d}]}{[\vec{a}\ \vec{b}\ \vec{c}]},\ y = \dfrac{[\vec{c}\ \vec{a}\ \vec{d}]}{[\vec{a}\ \vec{b}\ \vec{c}]},\ z = \dfrac{[\vec{a}\ \vec{b}\ \vec{d}]}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$\vec{d}\left[\vec{a}\ \vec{b}\ \vec{c}\right]+\vec{b}\left[\vec{a}\ \vec{c}\ \vec{d}\right] = \vec{a}\left[\vec{b}\ \vec{c}\ \vec{d}\right]+\vec{c}\left[\vec{a}\ \vec{b}\ \vec{d}\right]$

Also, $\vec{d} = x(\vec{a}\times\vec{b})+y(\vec{b}\times\vec{c})+z(\vec{c}\times\vec{a})$

dot with $\vec{d}$ :

**Sol.29** Let $\bar{F} = x\,\bar{a}_1 + y\bar{a}_2 + z\,\bar{a}_3$

$3\bar{b}_1 - \bar{b}_2 + 2\bar{b}_3 = x\,\bar{a}_1 + y\bar{a}_2 + z\bar{a}_3$

$3 = 2x + y + 3z$ ...(i)

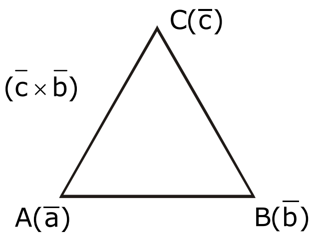
$-1 = 3x - 2y - z$ ....(ii)

$2 = -x + 2y - zz$ ....(iii)

Solve (1), (2) & (3)

$x = 2 \quad y = 5 \quad z = 3$

**Sol.30** (i) $\overline{AB}\times\overline{AC} =$




$((\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{b})$

$\bar{p}.\bar{n} = \bar{a}.\bar{n}$

$\bar{p}.(\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}) = [\bar{a}\ \bar{b}\ \bar{c}]$

(ii) Let $\bar{v} = \lambda\left(\overline{AB}\times\overline{AC}\right)$

$\bar{v} = \lambda\ (\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})$

dot with $\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}$

& use the relation from (1) part (1)

$[\bar{a}\ \bar{b}\ \bar{c}] = I+|2\Delta|2$

$\lambda = \dfrac{[\bar{a}\ \bar{b}\ \bar{c}]}{4\Delta^2}$

$\bar{v} = \pm\dfrac{[a\ b\ c](\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})}{4\Delta^2}$

**Sol.31** (a) take dot with $\bar{a}$ $p(\bar{x}\,.\,\bar{a}) = \bar{a}\,.\,\bar{b}$

Cross with $\bar{a}$

$p(\bar{x}\times\bar{a}) + (\bar{x}\times\bar{a})\times\bar{a} = \bar{b}\times\bar{a}$

$P(\bar{b} - p\,\bar{x}) + (\bar{x}\,.\,\bar{a})\,\bar{a} - |a|^2\ \bar{x} = \bar{b}\times\bar{a}$

$p\,\bar{b} - p\,\bar{x} + \dfrac{\bar{a}.\bar{b}}{p}\,\bar{a} - |a|^2\ \bar{x} = \bar{b}\times\bar{a}$

$\bar{x} = \dfrac{p^2\bar{b}+(\bar{a}.\bar{b})\bar{a}-p(\bar{b}\times\bar{a})}{p(p^2+a^2)}$